

STANDARD 16

EXCELLENCE AND EQUITY FOR ALL STUDENTS

Introduction

This overview discusses excellence and equity and how they are described in the *Mathematics Standards* of the New Jersey State Department of Education's *Core Curriculum Content Standards*. It is followed by five sections entitled *High Expectations in Mathematics for All Students*, *The Significance of Mathematics*, *Overcoming the Barriers to Equity*, *Challenging All Students to Maximize Their Achievement*, and *Identifying Equity Concerns in Districts and Schools*.

All students will demonstrate high levels of mathematical thought through experiences which extend beyond traditional computation, algebra, and geometry.

Descriptive Statement

High expectations for all students form a critical part of the learning environment. The belief of teachers, administrators, and parents that a student can and will succeed in mathematics often makes it possible for that student to succeed. Beyond that, this standard calls for a commitment that all students will be continuously challenged and enabled to go as far mathematically as they can.

High Expectations for All Students

The first fifteen standards set out high expectations for all students, and this final standard insists that all students **need to**, **can**, and **will** meet those standards.



Some kids
just can't
learn math.

THAT'S A MYTH.
All kids can
learn math; it doesn't
take special ability —
just persistence and
enthusiasm.





Math doesn't
run in my
family.

THAT'S A MYTH.
Children don't
inherit their
parents'
difficulties
with math.



As discussed in the Introduction to this *Framework*, our students **need to** meet these standards in order for them to be well prepared for careers in the 21st century, and in order for our state and country to have suitable employees in the 21st century.

But **can** all students meet high standards? Our answer is unequivocally *Yes*. New Jersey's *Mathematics Standards* sets high standards for all students, and at the same time insists that those standards are indeed attainable by all students. There will undoubtedly be exceptions, but those exceptions should be exceptional.

Achieving these standards for all students will not be easy, but it must start from the assumption that every individual student can achieve these standards. There are many barriers that have to be dealt with, and these are discussed later in this chapter, but none of these barriers are more powerful than the pervasive sense that many of our children cannot achieve these standards.

We must take seriously the goal of preparing ALL students for twenty-first century careers. In order to do this, we must overcome the all too common perception among people that many students simply lack mathematical ability. *Everybody Counts*, a report prepared by the Mathematical Sciences Education Board (MSEB) of the National Academy of Sciences (1989), notes the following:

Only in the United States do people believe that learning mathematics depends on special ability. In other countries, students, parents, and teachers all expect that most students can master mathematics if only they work hard enough. The record of accomplishment in these countries — and in some intervention programs in the United States — shows that most students can learn much more mathematics than is commonly assumed in this country (MSEB, 1989, 10).

Curricula that assume student failure are bound to fail; we need to develop curricula which assume student success. We need to develop attitudes in students, in parents, and in school personnel which assume student success. And we need to translate those positive attitudes and high expectations into programs which ensure that students **will** meet the standards. This chapter is intended to provide information and guidance to districts on how to make that translation.



Girls are just
not good
at math.

THAT'S A MYTH.
All children,
girls and boys,
are equally good
at math.¹



Opportunities to Exceed the Standards

By insisting that all students can meet high expectations, the *Mathematics Standards* does not imply that all students will become professional mathematicians, scientists, or engineers. Certainly, as with other areas of human endeavor, some students have more interest in and talent for mathematics than others.

The *Mathematics Standards* does not describe expectations for those students who might be going on to careers which require higher levels of mathematics. Its focus is on the high expectations which are appropriate for all students. However, the *Mathematics Standards* insists that raised expectations for all students should not result in lowered expectations for our high achieving students.

Indeed, to increase the number and success of high achieving students it is necessary to provide all students with opportunities to learn more mathematics than is encompassed in the *Mathematics Standards*. This is discussed further in the section entitled ***Challenging All Students to Maximize Their Achievement***.

What is Equity?

All students need to achieve, and can achieve the high expectations of New Jersey's *Mathematics Standards*. Ensuring that all students have every opportunity to achieve these high expectations is the focus of our concerns about equity.

The United States has traditionally promoted education as the most effective vehicle for access to intellectual development and economic independence. The *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM, 1989) cites data to support this premise. For example, jobs requiring mathematical knowledge and skills in such areas as data analysis, problem solving, and statistics are growing at nearly twice the rate of overall employment.² In addition, the strongest predictor of earnings nine years after graduation from high school is the number of mathematics courses taken (after having taken into account demographic factors) (NCTM, 1992, 3). In order to compete effectively in today's global, information-based economy, and in today's increasingly high-tech work

¹ Drawings by Mira Rosenstein.

² *Making Points for Mathematics Education Reform*, the Mathematical Sciences Education Board, p. 3.

environment, students must be able to reason logically, solve problems and communicate effectively. Clearly they must be educated at a more sophisticated level of mathematical and scientific reasoning than ever before. Mathematics and science, therefore, have been called the “critical filters” for determining future success.

Achievement in mathematics, science, and technology, however, has not been equally accessible to all students. Certain ethnic and racial minorities in the United States, including populations of students from economically disadvantaged backgrounds, are substantially *underrepresented* among top achievers, and are included in disproportionate numbers among those whose achievement is unsatisfactory. Young women are still underrepresented in these disciplines, as compared to young men.

According to a statement developed by the Equity Focus Group for the Statewide Systemic Initiative supported by the National Science Foundation (1994) “equity means equitable access to high-quality science, mathematics, and technology education and equitable treatment in the classrooms, schools, and post-secondary education institutions for every student. The goal is to eliminate the academic performance gap between mainstream groups and underrepresented groups, and to raise the level of knowledge and skills in mathematics and science of all students.”³ This goal is reflected in Standard 16 of the New Jersey’s *Mathematics Standards*.

A recent study by the Rand Corporation⁴ confirmed just such improvement in the last fifteen years: standardized test scores for African-American and Latino teenagers improved significantly between the mid-1970’s and 1990, narrowing the gap with Caucasian students, who also made gains. The Rand study said that “the gains suggested that desegregation and increased spending, especially for programs designed for minority students, had paid off.”

Each district should commit itself to reducing the gap between its own mainstream and underrepresented groups, without diminishing the performance of the mainstream groups. The final section of this chapter, ***Identifying Equity Concerns in Districts and Schools***, provides suggestions for how a district or school might evaluate its own situation with regard to this goal.

³ National Science Foundation Statewide Systemic Initiative. (1994). *Equity Framework in Mathematics, Science, and Technology Education*. Washington, DC.

⁴ Grissmer, D. W. (1994). *Student achievement and the changing American family*. Santa Monica, CA: Rand Corporation.

Cumulative Progress Indicators

New Jersey's *Mathematics Standards* provides the following eleven cumulative progress indicators for Standard 16. They are arranged in four groups, and their implications are discussed in the next four sections of this chapter.

The first two indicators address our commitment to high expectations in mathematics for all students, that all children learn mathematics in accordance with the vision of the *Mathematics Standards*.

By the end of grade 12, all students:

- 1. Study a core curriculum containing challenging ideas and tasks, rather than one limited to repetitive, low-level cognitive activities.**
- 2. Work at rich, open-ended problems which require them to use mathematics in meaningful ways, and which provide them with exciting and interesting mathematical experiences.**

In order for children to value mathematics, they need to understand the importance of mathematics in their own culture and other cultures, they need to understand that the quantity and quality of their own mathematical achievements will affect their futures, and they need to know that members of their community use mathematics in their own occupations. The following three indicators address these issues:

By the end of grade 12, all students:

- 3. Recognize mathematics as integral to the development of all cultures and civilizations, and in particular to that of our own society.**
- 4. Understand the important role that mathematics plays in their own success, regardless of career.**
- 5. Interact with parents and other members of their communities, including men and women from a variety of cultural backgrounds, who use mathematics in their daily lives and occupations.**

Children need to hear the message that all students can learn mathematics and that their schools are making a commitment to their achieving the high expectations in the New Jersey's *Mathematics Standards* through successful completion of the core curriculum. The following three indicators address these issues:

By the end of grade 12, all students:

- 6. Receive services that help them understand the mathematical skills and concepts necessary to assure success in the core curriculum.**
- 7. Receive equitable treatment without regard to gender, ethnicity, or predetermined expectations for success.**
- 8. Learn mathematics in classes which reflect the diversity of the school's total student population.**

Finally, all students should be provided with encouragement and opportunities to go beyond the expectations of the *Mathematics Standards*. The following three indicators address this issue:

By the end of grade 12, all students:

- 9. Are provided with opportunities at all grade levels for further study of mathematics, especially including topics beyond traditional computation, geometry, and algebra.**
- 10. Are challenged to maximize their mathematical achievements at all grade levels.**
- 11. Experience a full program of meaningful mathematics so that they can pursue post-secondary education.**

High Expectations in Mathematics for All Students

Engaging and Challenging All Students

The first two indicators specify that students should “study a core curriculum containing challenging ideas and tasks, rather than one limited to repetitive, low-level cognitive activities” and that they “should work at problems which require them to use mathematics in meaningful ways.” The vision of the *Mathematics Standards* (see pp. 8-9 of this *Framework*) speaks of “students who are excited by and interested in their activities”, “students who are learning important mathematical concepts,” and “students who are posing and solving meaningful problems.”

Students need to be engaged and challenged. To accomplish this, we need to involve them in hands-on activities, to provide them with settings where they can participate in mathematical discovery, to decrease the focus on repetitive tasks, to make available alternate ways of learning concepts, and to offer them activities which they recognize as meaningful.

Above all, we need to challenge them with meaningful problems. Indeed, problem solving should be the central focus of all mathematics instruction. Students benefit from a classroom environment in which they are working together to find solutions for meaningful problems. Levels of engagement, communication, and achievement are all enhanced when students accept these kinds of challenges. Moreover, such an instructional approach also tends to work more effectively than any other with heterogeneously grouped students. Students tend to find ways to contribute to the overall group progress by sharing their own skills and understandings with their classmates.

Traditionally, problem-solving oriented approaches to mathematics instruction have been used with only special populations of students. Remedial teachers frequently relied on such an approach to recapture the students’ lost interest or to rekindle their motivation to learn the subject. At the other end of the spectrum, problem-solving also frequently form the basis for an elementary school program for gifted-and talented students. Some of the best available curriculum units in elementary mathematics were written to be used with a gifted population. It was thought that these units were necessary to fully engage bright students and to illustrate for them the power and pervasiveness of mathematics.

But we must now focus on making this curricular focus the focus for all students. For just the same reasons that teachers thought this approach to be useful for special populations, it turns out to be useful for all.

Motivation, engagement, and appreciation for the usefulness and power of mathematics are dispositions that enable all students to be effective learners.

Core Curriculum

As noted on p. 12 of this *Framework* and reflected in Indicators 1 and 6, implicit in the vision of the *Mathematics Standards* is the notion that there should be a core curriculum. What do we mean by a “core curriculum”? We mean that every student will be involved in experiences addressing all of the expectations of each of the other fifteen content standards. We anticipate that over time each district will review the *Mathematics Standards* and, using this *Framework* as a guide, will develop its own core curriculum. All courses of study in the district should then have a common goal of completing this core curriculum, no matter how students are grouped or separated by needs and/or interests.

A core curriculum does not mean that all students will be thrown together into one program. There may be different programs with different goals, but completing the core curriculum should be a goal that is common to all of the programs. Students have different aptitudes, interests, educational and professional plans, and learning styles. Different groups of students may address the core curriculum at different levels of depth, and may complete the core curriculum according to different timetables. Nevertheless, all students should complete all elements of the core curriculum recommended in the *Mathematics Standards*.

For example, it is anticipated that those students who normally go on to take calculus in the 12th grade will complete the core curriculum, and go substantially beyond it, by the end of 10th or 11th grade. Indeed, at the present time, with curricula currently in place, those students are likely to complete most of what would be in the core curriculum, and more, by the 11th grade, with the major exception of the content described in Standards 12 (Probability and Statistics) and 14 (Discrete Mathematics). On the other hand, a substantial percentage of the students in most districts might well be involved with the core curriculum through the end of the 12th grade.

At the elementary school level, each district already has in effect a core curriculum which all students are expected to complete, but it may be focused primarily and even exclusively on arithmetic; however, the *Mathematics Standards* recommends a core curriculum with more substantial expectations than the current curricula. In addition, the recommendation of a core curriculum at the high school level has major implications for the elementary school level, since every student needs to enter the upper grade levels with both the knowledge and the confidence to achieve the expectations of the *Mathematics Standards* during her or his remaining years in school.

The core curriculum recommended in the *Mathematics Standards* is appropriate also for students in vocational education. According to the Secretary’s Commission on Achieving Necessary Skills (SCANS), a study of competency in the workplace across the *entire* spectrum of the economy revealed a clear pattern of requirements. Workers need a solid foundation in retrieving, analyzing, and evaluating data, applying technology to specific tasks, and the ability to reason, think creatively, and solve problems.⁵ Clearly this is the case for students involved in vocational education programs as well as college preparatory programs, and fulfilling the core curriculum of the *Mathematics Standards* would address the SCANS recommendations.

⁵ *Learning a Living*, SCANS (April 1992, 5-6).

High Expectations in the Elementary and Middle School — The Issue of Grouping

In the abstract, it seems to make sense to group students according to ability; higher ability students can proceed more rapidly, and lower ability students can receive additional instruction. In practice, however, homogeneous grouping of students (i.e., where students of similar abilities are grouped together), and its logical extension of tracking students into entire programs based on selected abilities, has limited the achievement of a substantial percentage of children. The research shows that once placed in a track, there is little chance of moving to a higher track; that tracking in mathematics is often based on reading scores or poor behavior rather than on mathematical ability; and that a disproportionate percentage of low-income and minority students are placed in low tracks. Students get the message that less is expected of them.

Moreover, the practice of homogeneous grouping and tracking of students is based on the premise that the abilities of students are substantially different. In fact, exceptional children are indeed the exceptions; all students have the ability to succeed in mathematics and to be empowered to use it successfully. “Individual differences in ability are not great enough to warrant differences in curriculum, except in unusual circumstances such as major learning disabilities or extraordinary talent,” said leading mathematics educator Zalman Usiskin.⁶ The reason some students appear to have high ability is often because they have been better prepared, they are building on a foundation of greater knowledge, and they have greater interest and willingness to work.

Heterogeneous grouping (i.e., where students who appear to have different abilities are grouped together) reinforces the message that all students can succeed and can meet high standards. The research shows that heterogeneous grouping does have the desired effect: all students’ self-image and self expectations rise, as does their performance (SCDOE, 1992, 26).⁷

What happens in the first few years of school is crucial. If all children in the elementary grades develop mathematical competency and positive attitudes toward mathematics, then equity problems will be less significant later on. If we want all students to complete the elementary levels with the knowledge and confidence that are needed for success in the upper grades, then heterogeneous grouping must be a key component of our strategic plan.

Heterogeneous grouping by itself, however, will not ensure success. It is a strategy, not a solution. The focus must be on providing all children with opportunities to learn, with encouragement to succeed, and with continued mathematical growth. In some settings, the strategy of heterogeneous groups may be counterproductive — resulting in some children experiencing only the frustration of failure, and others the boredom of stagnation. This may often be the case because teachers have not been prepared to implement this strategy. Training and support for teachers are critical; they must be in place before and during implementation of heterogeneous grouping, so that teachers can respond flexibly to the diverse needs of the students in the classroom. Teachers must be prepared to function as problem-solvers, ready to use a variety of strategies to ensure that all children learn; they must be familiar with these strategies, and understand when they should be used, and when they should not be used. We do our children a disservice if we abandon the old dogma of homogeneous grouping only to adopt a new dogma of heterogeneous grouping; we do our

⁶ *Individual Differences in the Teaching and Learning of Mathematics*, presentation at the Ninth Annual UCSMP Secondary Conference, November 1993. See also *If Everybody Counts, Why Do So Few Survive?* in *Reaching All Students with Mathematics*, NCTM, 1993, Reston, VA.

⁷ South Carolina Department of Education. (1992). *South Carolina Mathematics Framework*. Columbia, SC.

children a greater disservice if we adopt heterogeneous grouping without assisting teachers to implement it properly. Administrators must ensure that this does not happen; they must encourage teachers to learn how to teach children who are grouped heterogeneously, and how to strike an appropriate balance between that and homogeneous grouping. The rationale for any form of grouping must be reflected in the structure of the classroom, in the activities that take place there, and in the instructional strategies used by the teacher. We suggest that:

✓	<i>SUGGESTIONS</i>
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- ✓ districts use heterogeneous grouping as a strategy, at least through 6th grade, and possibly in 7th and 8th grades depending on local circumstances.
- ✓ districts provide opportunities for continuous and rigorous professional development to its teachers to help them develop the variety of instructional strategies needed for effective student learning, with a focus on heterogeneous grouping and how and when it can best be used.
- ✓ districts use heterogeneous grouping as an opportunity to provide an important advantage for the better prepared and highly motivated students — the opportunity to strengthen their own understanding through sharing it with others.
- ✓ districts provide assistance to those students who require it, enrichment for all students, and additional opportunities for those who are better prepared and highly motivated.
- ✓ since students progress at different rates, schools use flexible strategies (including enrichment activities) that ensure that all students have the opportunity to learn when they are ready.
- ✓ teachers closely monitor student performance and provide assistance to those students who need it. Priority should be placed on preventing students from falling behind through:
 - * additional instructional sessions;
 - * individual assistance in class;
 - * tutoring outside of class;
 - * cross-age tutoring in after-school programs;
 - * after-school access to technology;
 - * activities between school sessions;
 - * summer sessions; and
 - * enlisting support and assistance from adults in parental or community support roles.

High Expectations in the Secondary School

Each district is expected to develop a core curriculum based on the *Mathematics Standards* which embodies high achievable expectations for all students. In developing its core curriculum, districts should consider the following suggestions:

✓	<i>SUGGESTIONS</i>
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- ✓ districts may institute different courses and programs for different groups of students, but all of these should have a common goal — completing the core curriculum.

- ✓ the core curriculum should not be simply a rearrangement of traditional mathematics topics taught traditionally, which in the past has served to filter out substantial numbers of students, but should focus on making mathematics, including both traditional topics (such as algebra and geometry) and new topics (such as probability and discrete mathematics) relevant, exciting, challenging, and accessible to all students.
- ✓ all general mathematics, consumer mathematics, and other courses which focus primarily on mastery of lower-level basic skills should be transformed into courses which address the core curriculum as well as basic skills.
- ✓ districts should maintain support systems and enrichment opportunities that appropriately address the needs of their students.
- ✓ students should be permitted and enabled to cross over from one sequence of courses to another by demonstrating mastery of the material.
- ✓ all students, regardless of their backgrounds and abilities, should have the opportunity to study this core curriculum in an environment which allows them to develop to their fullest potential. Such an environment is one where⁸:
 - * students are encouraged to communicate freely and to take risks in asking mathematical questions and proposing solutions to mathematical problems, and feel safe in doing so;
 - * teachers recognize that students have the potential to learn mathematics at a high level;
 - * instructional strategies accommodate the cultural diversity, varied learning styles, and different amounts of time needed by different students;
 - * placement decisions are made in the interest of elevating students to the most challenging course they can be successful in;
 - * placement decisions are based on multiple assessment measures and recommendations of classroom teachers;
 - * classroom assessment includes continuous evaluation of students' understanding and performance using a variety of assessment strategies;
 - * all students have the opportunity to study all of the core mathematics curriculum;
 - * students are expected to do complete work by thinking, drawing on mathematical ideas, and using appropriate tools and techniques;
 - * students are expected to communicate their reasoning and problem-solving strategies and are able to hear and discuss different strategies used by other students;
 - * students are encouraged to work harder, and are provided with the necessary support if they are having difficulties;
 - * students with a special interest in mathematics pursue issues with similarly interested peers, while still participating in the common core curriculum;
 - * students have sufficient time to process questions, formulate their answers, and present them to the class;

⁸ This list is adapted and extended from a similar list in the Field Review Draft (1992) of the *South Carolina Mathematics Framework*, which in turn based its list on the draft (1991) *Mathematics Framework for California Public Schools*, 51.

- * students have a chance to revise and resubmit their work until it meets quality standards;
- * students have access to the same quality of technology;
- * the dynamic of students teaching students is harnessed through grouping strategies to leverage the teacher's instruction; and
- * services are available to help students who have gaps in their skills and understandings.

The Important Role of Mathematics

In order to achieve Indicators 3-5, students need to understand the importance of mathematics in their own culture and in other cultures, they need to understand that their futures will be affected by their mathematics achievement, and that mathematics is indeed used by adults in their communities.

Mathematics in All Cultures

Mathematics plays an integral role in art, music, games, explorations, inventions, and commerce within virtually any culture. People in all societies have devised their own ways of doing mathematics, and an inclusive study of cultures and their various contributions to mathematics is an effective way to demonstrate its relevance to all students^{9 10}. We suggest that teachers:



✓ ask groups to research the role of mathematics in various fields of human endeavor, and to report to the entire class on their projects.

✓ design instructional units that help students experience how decisions involving mathematics are made in relation to the needs and practices of various cultures. Such discussions can be used to connect mathematics with other content areas such as history, literature, sociology and art. For example, when designing rugs or quilts (see vignette), students are using the mathematics of geometry and measurement. A broader topic is the use of patterns in various cultures — for example, in art or in clothing. Another topic might be the types of architecture which characterize various times and places. For instance, a study of housing might revolve around the concepts of size, shape, perimeter, and area, and may be used to develop skills of estimation and approximation. Types of housing might include an African round house, a tipi, an urban apartment, and a suburban ranch house. (NCTM, 1993, 54).

⁹ For information about mathematics and culture, contact the International Study Group on Ethnomathematics, Math-Tech, 9155 N. 70th Street, Milwaukee, WI 53223.

¹⁰ Some helpful sources are *Multicultural Mathematics Materials*, Marina Krause, NCTM, Reston, VA; *Ethnomathematics: A Multicultural View of Mathematical Ideas*, Marcia Ascher, Brooks/Cole Publishing Company, Pacific Grove, CA; and *Africa Counts: Numbers and Patterns in African Culture*, Claudia Zaslavsky, Warren Mill Books.

Mathematics, Folk Art and Literature: Making Connections

In a third grade classroom, students planned and then created a quilt as a culminating experience. As a class, the children used measurement to divide the quilt into separate squares. Decisions about symmetry, alignment, design, and effective use of space were addressed by the whole class. Then each child designed one quilt square which represented an important piece of learning they valued during the year. Next, students, working in cooperative groups, organized the separate quilt squares into thematic groupings. Children looked for patterns among the separate quilt squares, proposed several ways to organize the squares and reached consensus. Earlier, the children read several fiction and nonfiction texts about quilting as a folk art. The finished quilt was displayed prominently at the Board of Education for the entire school community to view.

- ✓ review the counting words in various languages and analyze the different schemes for counting; in some cultures, for example, grouping is done by twenties, rather than by tens (NCTM, 1993, 54).
- ✓ use materials that reflect the diversity of cultural backgrounds of the students in the classroom or school.
- ✓ include history of mathematics in daily lessons, including examples of male and female mathematicians and scientists from a variety of cultures.
- ✓ be alert to the possibility that cultural perspectives may have the effect of discouraging some students from succeeding in mathematics.

Mathematics in Their Future

Students need to be aware that the mathematics that they learn, the problem-solving that they do, will affect their futures. One of the problems described in the New Jersey State Department of Education's *Directory of Test Specifications and Items*¹¹ asks students to determine a schedule for employees at a fast food place so that each employee has a reasonable schedule, so that sufficient staff is available at peak times, and so that labor costs are minimized. Having the problem-solving skills to tackle this kind of problem will differentiate between those who are eligible for managerial positions and those who be unable to advance beyond minimum-wage positions. This point is also made by the National Action Council for Minorities in Engineering's (NACME) "Math Is Power" video public service announcement and print materials¹².

Students need to be aware that mathematics is used by artists and musicians, by scientists who are involved in space travel, and by designers of skateboards. This point is stressed in the following materials:

- *Mathematics: Making a Living, Making a Life*, a booklet demonstrating how mathematics is all around us, and *Mathematics: Making the Connection*, a video which shows viewers how a jazz musician, architect, and newspaper publisher connect mathematics to their daily lives and professions; available

¹¹New Jersey State Department of Education. *Directory of Test Specifications and Items — 1991 Grade 11 High School Proficiency Test*. Trenton, NJ, 1993.

¹²The National Action Council for Minorities in Engineering (NACME) is dedicated to expanding minority participation in engineering and the sciences. NACME can be contacted at 3 West 35th Street, New York, NY 10001, by phone at (212) 279-2626, or via their World Wide Web site <http://www.nacme.org>.

from the National Council of Teachers of Mathematics (NCTM)¹³.

- *SETQuest Career Discovery* video, CD-ROM, and print materials that explore careers that use science and mathematics with candid career profiles of “on-the-job” educational requirements for a variety of careers; available from The Consortium for Mathematics and its Applications (COMAP)¹⁴.
- *101 Careers in Mathematics*, biographical essays of individuals with careers that require a solid background in mathematics, and *She Does Math! Real Life Problems from Women on the Job*; available from the Mathematical Association of America (MAA)¹⁵.

A very important way in which teachers can reinforce this message is to have community members come to the school and share their biographies and their experiences. Visitors should include both those whose professions are obviously mathematical or scientific and those whose professions are not. In the first instance, the students will be impressed by their activities, and, in the second instance, they will be surprised at the problem-solving activities in which “ordinary” people are involved.

Overcoming Barriers to Equity

The third group of indicators focus on ensuring that all students are provided the opportunity to succeed in mathematics — that they receive the appropriate services and are treated equitably. In this section we discuss the variety of barriers to achieving equity in our schools and how those barriers can be overcome. These include the following types of barriers, each of which is discussed in a separate section below:

- attitudinal barriers — attitudes and beliefs of students, parents, and educators;
- gender barriers;
- economic, language, and disability barriers; and
- geographical barriers — special problems faced in urban and rural settings.

Overcoming Attitudinal Barriers to Equity

An important set of barriers to equity are the attitudes and beliefs of students, parents, and educators which influence student outcomes in mathematics.

Students’ **attitudes toward mathematics** are an important factor in their learning of mathematics; those who enjoy mathematics and have confidence in their mathematical abilities are more likely to succeed. Studies have shown that minority students have positive attitudes toward mathematics in the primary grades. Like other students in the United States, however, they become less positive about mathematics as they proceed

¹³The National Council of Teachers of Mathematics (NCTM) catalog of materials can be requested by calling (703) 620-9840 or by e-mail orders@nctm.org.

¹⁴The Consortium for Mathematics and its Applications (COMAP) catalog of materials can be requested by calling 1-800-772-6627 or by e-mail order@comap.com.

¹⁵The Mathematical Association of America (MAA) catalog can be requested by calling 1-800-221-1622.

through school; both confidence and enjoyment of mathematics decline as students move from elementary through secondary school (NCTM, 1993, 22)¹⁶. Similarly, although girls do better in mathematics than boys in the early grades, a major drop in their self-confidence, and a concomitant decline in their performance, occurs in the fifth and sixth grades.

Students' **beliefs about mathematics** are also an important factor. Those for example who believe that all mathematics problems should be solved quickly will curtail their efforts if they encounter problems which take more than a few minutes, or if they find that a problem takes them more time than other students. Similarly, we have to be aware that all students experience both positive and negative emotions as they learn mathematics, and that students will develop negative beliefs about their mathematical abilities if their negative emotions are not balanced with positive emotions about mathematics. Learning mathematics is not always easy, and will at times generate frustration, but we must ensure that our students' experience of frustration is not converted into a sense of failure.

Another important factor is students' **beliefs about the utility of mathematics**. Majority students, in general, rate mathematics higher in utility than do minority students (The Mid-Atlantic Equity Center, 1992, 9)¹⁷. Males in general perceive mathematics as more useful and valuable to their futures than do females. One significant consequence of these differences in attitudes is that males are therefore more likely to take more mathematics courses and, as a result, have more desire and more opportunity to pursue mathematics-related careers.

Also important is **how students account for their success or failure in mathematics**. As noted earlier, in other countries success or failure in mathematics is usually attributed to hard work or its absence, whereas in the United States success or failure in mathematics is usually attributed to ability. *This is particularly true of female students, who tend to attribute their success to extra effort and their failures to lack of ability; males on the other hand, are more likely to attribute their success to ability and their failures to lack of effort.* Research on confidence in learning mathematics indicates that males tend to be more confident even when females have more reason (based on their achievement) to be confident.

Research indicates that **attitude and achievement interact with each other in subtle and often unpredictable ways**. For example, while students in Japan express a greater dislike for mathematics than students in other countries, they exhibit a very high level of proficiency in mathematics. More common here, however, is that identification as a low achiever, or placement in a lower track, has a negative impact on a student's self-confidence and belief in his or her own ability to learn mathematics.¹⁸

Finally, **peer pressure** is often a barrier to student achievement in mathematics. Students from underrepresented groups sometimes hold attitudes which are counter-productive and discourage their peers from achieving. Peer pressure to avoid academic excellence can be particularly difficult to combat among minority adolescents because they sometimes link it to majority cultural values or to disloyalty to their group (USDOE, 1993, 13).

¹⁶ National Council of Teachers of Mathematics. 1993. *Reaching All Students with Mathematics*. Reston, VA.

¹⁷ Mid-Atlantic Equity Center. (1992). *Opening up the Mathematics and Science Filters*. Washington, DC: The American University.

¹⁸ *Mathematics Framework for California Public Schools*, California Department of Education, 62.

Attitudes and beliefs which discourage achievement are not limited to students; parents, educators, and policy makers also exhibit reduced expectations for some students.

Parents and teachers of high achieving girls sometimes reduce their expectations when girls reach adolescence out of concern that high achievement may jeopardize social approval. One mother refused to allow her seventh grade daughter to enroll in an algebra program for which she had been selected. She explained, “The stress of such a high powered class is likely to make her acne worse. She doesn’t need that at her age.” In class, teachers call on girls less frequently than boys, and tend to have shorter interactions with them. Well-meaning teachers also sometimes limit gifted girls to protect their popularity. In one eighth grade class, the two gifted boys were sent to the high school for math, while the parents of the two girls, who had similar abilities, were told that their daughters would be better served by remaining in the classroom because “there’s a stigma for girls who go to the high school for math.”

Parents and teachers of minority students also frequently believe that their children cannot be successful in mathematics and try to protect them by placing them in lower level courses; policy makers try to protect them by arguing against challenging expectations. While it is very difficult for students to overcome inadequate elementary and middle school experiences with mathematics, there is no reason to believe that minority students with positive early experiences with mathematics will not be successful in high school.

Parents who themselves had negative experiences with mathematics tend to transmit their apprehensions to their children and communicate that lower expectations in mathematics are acceptable.

It’s a myth that some kids can do math and others just can’t. It’s a myth that girls are not as good at math as boys. It’s a myth that children inherit trouble with math from their parents. We need to encourage *all* children to succeed in mathematics.

We suggest that teachers, schools, and districts:

✓	<i>SUGGESTIONS</i>
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- ✓ use instructional strategies (see Chapter 17) that provide positive experiences for students and engender in them enjoyment in mathematics and confidence in their abilities to do mathematics.
- ✓ convey to students that it is normal to find mathematics frustrating at times and, through examples, that it sometimes takes a considerable amount of time to solve mathematical problems. (Students need to understand the difference between an exercise, which usually involves applying a simple specified procedure, and a problem, part of which involves determining what methods are appropriate.)
- ✓ maintain and convey high expectations for all children, and reinforce the message that all students can meet those high expectations.
- ✓ encourage students to strive for excellence in mathematics by providing opportunities for them to participate in math clubs, math teams, and other math activities¹⁹.
- ✓ reflect on your own attitudes toward students and your expectations for them by answering the

¹⁹ See footnotes 52-56 for information about these kinds of activities.

following self-reflective questions²⁰:

- * How do I interact with girls, minority students, and “low achievers” in class?
- * Do my interactions reflect lesser expectations for some groups than for others?
- * Do I believe that all students can learn mathematics?
- * Do I interact more with the “high achievers” and give less attention to “low achievers?”
- * Do I ask more questions and questions requiring higher levels of thinking to the “high achievers?”
- * Do I provide sufficient wait time for all students to formulate responses to questions?
- * Do I seat “high achievers” more closely to myself than “low achievers?”
- * Do I praise “high achievers” more often than “low achievers?”
- * Do I provide detailed feedback to “high achievers” and provide less precise feedback to “low achievers?”
- * Do I demand more work and effort from “high achievers” and accept less work and work of a lower standard from “low achievers?”
- * Do I provide a positive classroom environment in which students are willing to share their questions and their answers without fear of being blamed or shamed?

✓ provide effective professional development for teachers who lack sufficient content knowledge, or who have a limited instructional repertoire.

✓ use positive peer influence to help shape the attitudes and beliefs of students. Peer tutoring, especially between younger students and older peer tutors, can increase students’ interest and motivation.

✓ seek actively to dispel parental myths their children must inherit their anxieties and difficulties with mathematics.

✓ communicate with parents and encourage them to encourage their children to learn mathematics, even if their own experiences were not positive.

✓ utilize adult members of the community to help shape students’ attitudes about mathematics and help them recognize the usefulness of mathematics in their future careers. Female and minority members of the professional community who use mathematics in their daily jobs and lives can be enlisted to serve as role models for female and minority students and to help others recognize the contributions and possibilities of women and minorities.

✓ obtain information about how other schools and districts are addressing issues of equity²¹.

²⁰ Adapted from *Opening Up the Math and Science Filters*, MidAtlantic Equity Center, 1992, 1-7.

²¹ A good source is districts involved in the New Jersey Statewide Systemic Initiative (NJ SSI), “Achieving Excellence in Mathematics, Science and Technology Education” funded by the National Science Foundation and the New Jersey Department of Education. The NJ SSI is committed to the advancement of equity, and districts involved in NJ SSI can provide assistance in planning and implementation. For further information, call Roberta Schorr at (908) 445-2342.

- ✓ review and make use of the literature^{22 23 24} on equity in mathematics education.

Overcoming Gender Barriers to Equity

An equitable learning environment is one that encourages the mathematics development of male and female students from every cultural background. We suggest that teachers:

✓	SUGGESTIONS
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- ✓ self-assess your individual beliefs and habits in regard to gender-role stereotyping. Ask the following self-assessment questions^{25 26}:

- * do I believe females are less capable in mathematics than males?
- * do I allow male students to control the discourse?
- * do I interact more with male students than female students?
- * do I encourage risk-taking and autonomous behavior in female students?
- * do I believe it is more important for the boys to “get it?”

Check your responses by inviting a friend to tape your class and then analyzing your interactions on the tape with male and female students.

- ✓ avoid using the technique of increasing attention to and interaction with boys as a method for controlling the class. There is an apparent benefit to paying more attention to male students or allowing them to call out the answers, since it keeps the class on task and discourages disruptive activity. On the other hand, an important consequence is that it permits many female students to remain passive in their dealings with mathematics and invisible in the mathematics classroom.

²² The Consortium for Educational Equity at Rutgers University, New Brunswick has the nation’s most extensive library collection on equity in mathematics and science education. Subject bibliographies are created and updated on a regular basis to highlight the collection and to assist borrowers in reviewing materials they wish to use. Bibliographies on the following topics are particularly relevant: Cooperative Learning, Educational Equity: Training and Technical Assistance, Equity in Science and Mathematics, Futures Unlimited: Career Information, Futures Unlimited in Mathematics and Science (grades 7-12), Guidance and Counseling, Science-Related Storybooks for the Classroom, Science Stuff for Girls and Boys (grades K-6), Sources for Resources (for Librarians and School Library Media Specialists). In addition, subject bibliographies which support the understanding of the major cultural/ethnic groups in New Jersey have been developed for educators. Contact the Consortium at (908) 445-2071 to gain access to its collection.

²³The Mid-Atlantic Eisenhower Consortium for Mathematics and Science Education, located at Research for Better Schools, Philadelphia, has published a useful resource guide, *Equity Materials in Mathematics, Science, & Technology*, by Marylin A. Hulme. This guide includes print materials, resources for career information, and audiovisual materials for use with students and also in staff development programs. All the materials listed in this guide are included in the collection in the Resource Center at the Rutgers Consortium for Educational Equity, and are available for borrowing. For a copy, please call Research for Better Schools (215) 574-9300.

²⁴The Mid-Atlantic Eisenhower Consortium for Mathematics and Science Education maintains a World Wide Web Page dedicated to Equity Issues in Mathematics and Science. The address of this page is http://www.rbs.org/eisenhower/res_equity.html/. From this page you can link to a wide variety of equity sites.

²⁵ List adapted from *Teaching Mathematics Effectively and Equitably to Females*, Katherine Hanson, ERIC Clearinghouse on Urban Education: New York, NY, 1992, 31.

²⁶ See also the list referenced in footnote 20 which is adapted from *Opening Up the Math and Science Filters*.

- ✓ use language and materials that are free from gender-role stereotyping²⁷.
- ✓ provide career examples early in life to female students and encourage them to develop as mathematical thinkers. Research shows that girls do not think about advancing in mathematics because they have no idea of how they could use it in life (Hanson, 33).
- ✓ consider the following recommendations of the Institute for Urban and Minority Education²⁸:
 - * use situations that introduce girls and boys to a variety of mathematical-related career options;
 - * provide role models of men and women working in mathematics, technology, and the sciences;
 - * encourage female students as well as male students to participate in extracurricular math and science activities;
 - * be sensitive to the meanings of words — children do not translate “man” to mean both “man” and “woman”;
 - * encourage female students as well as male students to explore;
 - * introduce female and male students to action toys and activities such as team sports which increase their spatial visualization skills;
 - * make mathematics fun and appealing to both male and female students, using word problems that relate to the interests of both and that emphasize non-stereotyped roles;
 - * devise comfortable ways for students of both genders to play and interact;
 - * teach mathematics to young children through play; and
 - * discover early and correct promptly gaps in previous mathematical education so as to encourage both female and male students to continue in mathematics.

Overcoming Economic, Language, and Disability Barriers to Equity

Overcoming economic barriers to equity. More than 20% of the school children in the United States come from families in poverty (USDOE, 1991,1)²⁹. In New Jersey, more than 11.5% of school children are below the poverty line³⁰; of those who are in families with three or more children, over 20% are below the poverty line³¹. A greater percentage of these children experience failure at every level when compared to children from the middle class. Research points to three characteristics of the instruction these children typically

²⁷ Marylin A. Hulme of the Rutgers Consortium for Educational Equity has developed a series of questions for teachers and curriculum specialists to use when evaluating mathematics books for bias and stereotyping. These “guidelines” are intended to make the users aware of how the texts and illustrations depict girls and boys, women and men, the language used, and the roles that are assigned to both females and males. *The Guidelines for Evaluating Mathematics for Bias* can easily be used in conjunction with an evaluative chart/checklist. For more information, call (908) 445-2071.

²⁸ List adapted from *Teaching Mathematics Effectively and Equitably to Females*, Katherine Hanson, ERIC Clearinghouse on Urban Education: New York, NY, 1992, 31.

²⁹ United States Department of Education. (1991). *The Search for Effective Instruction of Children of Poverty*. Washington, DC.

³⁰ United States Department of Education. Digest of Education Statistics (1994), Table 20: Household Income and Poverty Rates. Based on information from United States Department of Commerce, Bureau of the Census.

³¹ New Jersey State Data Center, New Jersey Department of Labor. Table S6: Family Poverty Status in New Jersey. (November 1994). Based on information from United States Department of Commerce, Bureau of the Census.

experience that exacerbate problems with learning: *low expectations for what they can accomplish, misdiagnosis of their learning difficulties, and a failure of the schools to reexamine and restructure programs for these students* (USDOE, 1990, 6)³². What steps can we take to ensure equity for disadvantaged students? We suggest that teachers:

✓	<i>SUGGESTIONS</i>
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- ✓ provide instruction that utilizes students' prior knowledge, especially from real-work experience. Research indicates that minority students, for example, perform best in mathematics classes when the content is related to their previous experience (Mid-Atlantic Equity Center, 1992, 1-3). Thus, while recognizing the gaps in the students' formal knowledge, teachers need to build on students' experiential knowledge while at the same time expanding their knowledge base.
- ✓ provide mathematics instruction that is rich with problems and activities which connect mathematics to the everyday experiences of students. Activity-based programs have been demonstrated to significantly improve minority student performance in mathematics and science process skills (Mid-Atlantic Equity Center, 1992, 1-4). Supplement instruction with field trips to laboratories, college campuses, various worksites, and other similar places where it is apparent that mathematics is relevant and useful to the tasks at hand.

Overcoming language barriers to equity. Students who are not native English speakers are at a disadvantage in English-speaking mathematics classrooms and when taking tests constructed for English-speaking students. We suggest that teachers:

✓	<i>SUGGESTIONS</i>
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- ✓ use tools other than those that require English language proficiency in assessing the students' mathematical understanding and in making decisions about which mathematics courses should be taken by students who are not native English speakers. As their language skills improve, so will their performance in mathematics class.
- ✓ facilitate both learning of mathematics and proficiency in English³³ by
 - * designing and implementing activity-based programs in mathematics with built-in linguistic objectives;
 - * teaching mathematics as a component of bilingual programs;
 - * having students participate in purposely structured cooperative learning groups which will provide development in oral and written communication skills and enhance the student's academic self-image;
 - * instructing students in problem-solving strategies which include tools for decoding words and phrases; and
 - * presenting mathematics content in simplified or "sheltered" English. This has been shown to increase language competencies.

³² United States Department of Education. (1990). *The Search for Effective Instruction of Children of Poverty*. Washington, DC.

³³ Recommendations of the MidAtlantic Equity Center, 1992, I-4.

- ✓ make use of the literature on ESL instruction³⁴.

Overcoming disability barriers to equity. Research indicates that students with disabilities often have low academic self-images (The Mid-Atlantic Equity Center, 1992, 1-5). Neither special programs nor mainstreaming have been shown to significantly reduce these negative beliefs about the self. Poor self-image often persists from elementary through secondary school. Students with disabilities can master the curriculum content requirements for a high school diploma (NYDOE, 1994, 91)³⁵. Many can attend regular classes when provided with supplemental instruction and services. It is important, therefore, that such students receive instruction, from the elementary years on, in the core curriculum in mathematics recommended by the Standards. Following are some suggestions³⁶:

✓	SUGGESTIONS
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- ✓ enrich the mathematics program and classrooms by using technology in order to enhance learning experiences of students with disabilities³⁷.
- ✓ use a partner or assistant to work with the students with disabilities in the classroom as an additional resource to clarify classroom discussions and activities.
- ✓ encourage students to express their understanding, both with a partner and with the teacher.
- ✓ provide a suggested timeline and benchmarks for extended tasks and projects to students who have difficulty with organizational skills.
- ✓ use alternative testing strategies by modifying testing procedures and formats.
- ✓ align the mathematics and special education programs by providing teachers with collaborative planning time.
- ✓ provide professional development opportunities for special education teachers that further enhance their own mathematical backgrounds.
- ✓ provide special education teachers and students with video and audio tapes of classroom activities and discussions for further review.

Overcoming Geographical Barriers to Equity

Overcoming barriers to equity in urban schools. Students in urban schools present a special challenge for mathematics educators. Among the issues are:

³⁴ Some helpful sources are *Children and ESL: Integrating Perspectives* (TESOL, Washington, DC, 1986) and *When They All Don't Speak English: Integrating the ESL Student into the Regular Classroom*. (NCTE: Urbana, IL, 1989).

³⁵ New York Department of Education. (1994). *Curriculum, Instruction, and Assessment Framework for Mathematics, Science, and Technology*. New York, NY.

³⁶ List adapted from *Curriculum, Instruction, and Assessment Framework for Mathematics, Science, and Technology*, New York Department of Education (NYDOE), 1994, 91-92.

³⁷ Suggested resources are: *Directory of Resources of Technology for Special Education* and the May/June 1992 issue of *Technology and Learning*.

More time on task: Many urban students are behind, and getting farther behind every year. Thus one key element of an equitable curriculum is providing students with the classroom time required to catch up.

Engaging students: Additional instructional time alone is not sufficient. It must also be used well — just doing more of the same will not spark students’ interest or inspire achievement. We need to get beyond basic skills and present students with tasks, challenges and perspectives that are varied, interesting, and appropriate for employment and citizenship in the 21st century.

Contextual learning: Because urban students often have a very different experience base, standard classroom approaches that have been developed for students with middle-class backgrounds may seem foreign or contrived. We need to embed mathematical principles and learning within a range of projects and activities which students can identify as authentic.

Encouraging positive interactions: To overcome the negative peer pressure which devalues achievement in mathematics, we need to encourage students to have more positive and productive interactions in a mathematical context.

✓	SUGGESTIONS
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✓ provide opportunities for additional time on mathematics through additional instructional sessions, tutoring by teachers and students outside of class, tutoring by adults in the community, after-school access to technology, and summer sessions.

✓ select and design learning activities and projects which incorporate the use of mathematics in familiar contexts of interest to students. These could include locally important employment markets, the natural environment, educational puzzles and games, and sports or entertainment. Also appropriate are:

- * team-taught lessons or units with science teachers;
- * reports and projects jointly assigned and assessed with English teachers;
- * integrated lessons or projects with vocational teachers;
- * field trips to science museums and work sites;
- * use of technology to support both individual learning and group projects;
- * team-taught lessons with bilingual and ESL teachers, who sometimes are not aware of potential opportunities to enrich their lessons with mathematical perspectives; and
- * curricula specifically designed to incorporate hands-on mathematics laboratory activities such as *Applied Mathematics*³⁸.

✓ accommodate students with varying levels of proficiency in mathematics and monitor progress daily; the following practices³⁹ will help meet the needs of individual students:

- * start from where students are;
- * do not needlessly review;

³⁸ *Applied Mathematics* is a set of 36 modular learning materials prepared to help high school students and others develop and refine their job-related math skills. It was developed by a consortium of 41 states, in cooperation with the Center for Occupational Research and Development (CORD) of Waco, Texas.

³⁹ Zalman Usiskin (Director, University of Chicago School Mathematics Project), *If Everybody Counts, Why Do So Few Survive?* in *Reaching All Students with Mathematics*, NCTM, 1993, Reston, VA.

- * do not undermine students' self-confidence by placing them in courses for which they are unprepared;
 - * allow students of different ages to do the same mathematics together;
 - * provide remediation immediately and powerfully;
 - * use technology; and
 - * incorporate applications and real problem solving into the curriculum.
- ✓ determine the different learning style preferences of students, and design activities to accommodate a wide variety of learning styles.
 - ✓ use cooperative learning techniques to encourage students to interact positively and productively with each other.
 - ✓ use a variety of instructional styles and learning modes to capture and keep the attention of the students.
 - ✓ provide professional development to teachers about learning styles and instructional strategies and help them to implement their new knowledge in their own classrooms.
 - ✓ inform students about opportunities in higher education.
 - ✓ review Chapters 17, 18, and 19 for further information about instructional strategies, assessment, and technology.

Overcoming barriers to equity in rural schools. Because of their relatively small size and less adequate resources, many rural school districts cannot afford to have as broad a curriculum or as many specialized teachers as do wealthier districts. Moreover, they often offer a narrower range of courses and require individual teachers to cover more subjects than their larger counterparts. Rural students often are shortchanged when it comes to more advanced courses. The Children's Defense Fund (Sherman, 1992)⁴⁰ has reported, for example that (1) calculus was offered by approximately 33% of rural schools in the early 1980s, 50% of urban schools, and 67% of suburban schools, and (2) advanced placement classes — which offer talented students an important head start on earning college credit — were available in only 20% of the nation's rural schools in the early 1980s compared to nearly 50% of the suburban schools.

With regard to achievement, average rural achievement scores in most subjects are reported to be slightly below those in metropolitan areas and far below those in suburban areas. The Children's Defense Fund report cites a National Assessment of Educational Progress (NAEP) study conducted in 1981-1982 that provided a picture of students' mathematical skills based on the population size of the community where they attended school. Consistently students in the smallest communities and largest cities performed slightly below average in math. Students in medium-sized cities and suburbs performed somewhat better than average. These results suggest that communities at both extremes of population size face special problems in education.

In a more recent analysis of the assessment of student performance in rural schools⁴¹, the following data is

⁴⁰ Sherman, A. (1992). *Falling by the Wayside, Children in Rural America*. Washington, DC: Children's Defense Fund.

⁴¹ Stern, J.D., ed. (1994). *The Condition of Education in Rural Schools*. Washington, DC: US Department of Education, Office of Educational Research and Improvement.

reported: On the National Assessment of Educational Progress mathematics assessments in 1978 and 1982, the mean proficiency scores for rural students were below the national average. By 1986 and again in 1990, however, rural mean scores essentially matched the national average. The study also reported data on the National Education Longitudinal Study of 1988, indicating that rural 8th graders scored at about the national average on measures in mathematics, but that they scored significantly lower than their suburban counterparts.

✓	<i>SUGGESTIONS</i>
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The Children's Defense Fund recommended that states and school districts take a variety of steps to improve rural students' access to a broad range of programs and courses, including the following four points:

- ✓ all school districts should eliminate tracking and ensure that all students are taught a rigorous core curriculum.
- ✓ school districts should aggressively explore distance learning technologies that bring interactive classes to rural schools via satellite and interactive video; federal and state governments should promote distance learning, for example by expanding the Federal STAR schools program.
- ✓ states or consortia of schools should fund regional alternative schools or specialized magnet schools to increase the range of programs available to students.
- ✓ rural districts should not become distracted by issues such as consolidation of school districts with small populations. Consolidation is neither a magic answer, nor always a disaster. Communities must focus on the real need — good teachers in good facilities with good support.

Challenging All Students to Maximize Their Achievement

All students should be challenged to reach their maximum potential. For many students, the core curriculum recommended here will indeed be challenging. But if we do not provide this challenge, we will be doing our students a great disservice — leaving them unprepared for the technological, communication, and information age of the 21st century.

For other students, this core curriculum itself will not be a challenge. We have to make sure that we provide these students with appropriate mathematical challenges. We have to make sure that the raised expectations for all students do not result in lowered expectations for our high achieving students. A core curriculum does not exclude a program which challenges students beyond the expectations set in the Standards. Indeed, the *New Jersey Mathematics Standards* calls for all schools to provide opportunities to their students to learn more mathematics than is contained in the core curriculum.

Students who learn quickly, who have a high level of interest in mathematics, who are industrious and who are bored with repetition, are often under-challenged, and therefore, may not achieve their full potential. Most top students in the United States are offered a less rigorous curriculum, read fewer demanding books, complete less homework, and enter the work force or post-secondary education less well prepared than top

students in many industrialized countries (USDOE, 1993, 5).⁴²

Poor preparation in elementary and secondary school translates into student performance far below their potential. Only one-half of America's high-ability high school seniors from the class of 1980 (the top 25 percent as indicated by achievement tests) were estimated to have received a bachelor's degree by 1987, and only one in eight had entered a graduate program in any field by that date. In addition, U.S. students are not aspiring to, or are not qualifying for, our graduate programs in mathematics. In 1990, 57 percent of doctorates granted in the United States in mathematics went to students from other countries (USDOE, 1993, 11).

Most elementary and secondary school programs for students termed "gifted and talented" are often modest in scope. "The vast majority of talented students spend most of the school day in a regular classroom where little is done to adapt the curriculum to their special needs." The exceptional programs — specialized schools, magnet schools, and intensive summer programs — serve only a fraction of the secondary students who might benefit. Moreover, dual enrollment (where secondary school students also enroll in college) is uncommon (USDOE, 1993, 21-22).

Effective programs do, however, exist around the country; programs such as residential schools, summer activities, and enriched curricula are often aimed at developing analytical thinking skills in students, and often offer innovative approaches to scheduling and other organizational aspects of the mathematics program. In the past, these approaches generally "have not been implemented in regular education because educators did not realize their potential for improving all of American education. Now, however, many educators believe that the knowledge gained from these and other outstanding programs can be used to upgrade all of education."⁴³ (USDOE, 1993, 23).

Accordingly, the USDOE makes the following recommendations:

- * expand effective education programs and incorporate more advanced material into the regular school program;
- * provide all students with the opportunities to solve problems, analyze materials and situations, and learn from real-life experiences;
- * identify students who need individual or special opportunities, using test data only as appropriate;
- * serve especially talented students in many places: the regular classroom, special class, the community, at a university or a museum, in front of a computer, or anywhere the opportunity meets the need; and
- * create flexible schools that enable all students, including the most able, to be grouped and regrouped according to their needs and interests (USDOE, 1993, 24).

Assessment and Learning

⁴² United States Department of Education. (1993). *National Excellence: The Case for Developing America's Talent*. Washington, DC.

⁴³ The Accelerated Schools Project at Stanford University is a process based on whole school change which has shown remarkable results. When at-risk students are provided with the rich and varied approaches usually reserved for the gifted, their learning is accelerated. For further information, contact Dr. Henry M. Levin at (415) 723-0840 or at the Accelerated Schools Project, Stanford University, Stanford, California 94305.

A statewide assessment which assesses student understanding of the *Mathematics Standards* will be a major step in ensuring that all students indeed receive and learn the core curriculum. The New Jersey Department of Education is developing a fourth-grade statewide assessment aligned with the *Mathematics Standards*, called the Elementary School Proficiency Assessment (ESPA), and over the next few years will adapt the Eleventh-Grade High School Proficiency Test (HSPT) and the Eighth-Grade Early Warning Test (EWT) so that will continue to evolve to reflect the *Mathematics Standards*. The expectation is that these statewide assessments will reinforce the message of high achievable standards for all students and will support the vision of mathematics education reflected in the *Mathematics Standards*.

At the local level, we must move away from the notion that the major purpose of assessment is to filter out students, and move toward the notion that the major purpose of assessment is to improve learning. In order for assessments to serve this purpose, they must enable each student to demonstrate what he or she understands and is able to do. This is what equity implies in the context of assessment. This cannot be accomplished by any single instrument, and certainly not by one whose grading is only in terms of correct and incorrect. Assessments which are timed favor those who work best under those conditions, and assessments which are interactive favor those for whom interaction is an important component of learning. If the focus of assessment is improving learning, and not simply ordering students against a linear standard or preparing grades for a report card, then multiple options become available to the teacher.

Regular use of a variety of assessments, such as those discussed in Chapter 18, can provide the teacher and each student with specific information about his or her progress, so that efforts can be initiated, with both individual students and with entire classes, to address promptly any problems that students are having with the mathematical topics under discussion.

In discussing criteria for choosing an equitable set of assessments, one should consider how the assessments⁴⁴:

- * support the vision of New Jersey's *Mathematics Standards*;
- * enable students with diversity in experience and mathematical sophistication to respond to the assessments in ways that demonstrate what the students know and can do;
- * individually and collectively provide a composite picture of each students' understanding and skills;
- * communicate effectively to the student the mathematics problems being posed (using appropriate and multiple representations, including verbal descriptions, graphs, and other visuals);
- * support the learning of students who are bilingual or are developing their use of the English language;
- * encourage self-assessment and reflection leading to self-improvement, and motivate students to take responsibility for their learning;
- * occur under conditions (i.e., space, tools, and time) that enable each student to exhibit best work; and
- * seem reasonable for all students to complete in terms of outside school requirements, including access to libraries, technology, and other resources, affordability, and family and time constraints and responsibilities.

Community Involvement

⁴⁴ Adapted from the *Assessment Standards for School Mathematics*, National Council of Teachers of Mathematics (1993, 86-87).

Support from the community is essential for achieving equity and excellence. We must communicate to parents our message that all children must and can succeed in mathematics, and encourage them to communicate that message to their children. On the other hand, the community at large is a resource that can be tapped for services in extending students' mathematical experience beyond the school day and beyond the school curriculum. How then do we involve community members and parents? We suggest that teachers, schools, and districts:

✓	<i>SUGGESTIONS</i>
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- ✓ convey to parents the message that all children must and can succeed in mathematics⁴⁵, and encourage them to communicate that message to their children.
- ✓ convey to parents that even though they may have had negative experiences with mathematics when they were in school, their children need to be encouraged to succeed in mathematics.
- ✓ enlist parents, retired persons, and local business people to serve as mentors, tutors, translators, and encouragers.
- ✓ enlist guests from local business and industry to share their personal and professional biographies in the fields of mathematics and science.
- ✓ make arrangements for students to have adequate study facilities (in schools and in the community) and access to technology during after-school hours.
- ✓ arrange after-school study groups and encourage parents to exchange phone numbers so that students can review classwork and discuss homework.
- ✓ establish a homework hotline^{46 47}.
- ✓ sponsor activities for parents and students each April during Math, Science, and Technology Month⁴⁸ (and at other times of the year) which involve parents actively in mathematics and which inform them about the kinds of mathematical activities that are taking place in the school, and how they reflect national and state recommendations.
- ✓ provide opportunities for ongoing mathematics education for parents and other adults. Programs such as Family Math⁴⁹ can be initiated to increase parents' awareness of the need for mathematics and its role

⁴⁵ The New Jersey Mathematics Coalition has produced a guide for parents, *Mathematics to Prepare our Children for the 21st Century: A Guide for New Jersey Parents*. This document can serve as support for your message, and as an outline for a group discussion. Call (908) 445-2894 to purchase single or multiple copies of the revised Parents' Guide.

⁴⁶ The Richard C. Crockett Middle School (Hamilton Township, Mercer County) has had a homework hotline since 1990. Call (609) 890-3800 for information.

⁴⁷ The Extra Help Homework Hotline is a live call-in program for Newark students, which airs each Tuesday on Channel 3 (Cablevision) from 3:30 to 5:30 p.m. (with taped repeat on Wednesday from 8 to 10 p.m.). For information call Kenneth Herskovits, Central High School, at (201) 733-8656 or, in the evenings, at (212) 733-8656.

⁴⁸ For additional information about Math, Science, and Technology month contact: The New Jersey Mathematics Coalition, P.O. Box 10867, New Brunswick, NJ 08906 or call (908) 445-2894.

⁴⁹ Family Math is an innovative parental involvement program which provides parents and children in grades K-8 with opportunities to build understanding of mathematical concepts using inexpensive hands-on materials such as beans, toothpicks, and coins. The teaching emphasis is on doing mathematics; time is not spent on worksheets or in a lecture format. Children and adults come together once a week for six weeks to work cooperatively, learn to reason and think

in shaping the future successes of their children. Family Math offers activities for parents and their children which heighten children's interest and encourage parental support for mathematics.

- ✓ reach out to parents who appear nonsupportive and who do not participate by conducting activities where parents normally congregate, such as churches or community rooms in housing projects, and by offering a wide variety of opportunities for involvement which require a low level of commitment, such as establishing friendly home to school communications.
- ✓ encourage parents to engage their children in extra-curricular mathematics projects in the home⁵⁰

EQUITY 2000⁵¹, a project of the College Board, has as its goal to close the gap in the college-going and success rates between minority and non-minority students, and advantaged and disadvantaged students by proposing academic excellence for all students. This is accomplished through:

- * creation of district-wide policy changes to end tracking and raise standards for all students, beginning with the requirement that all students complete algebra by the ninth grade and geometry by the tenth grade, and including reform of the curriculum to reflect standards set by the National Council of Teachers of Mathematics and other discipline-based organizations;
- * establishment of ongoing professional development for teachers, counselors, and principals to increase their professional knowledge and skills and to raise their expectations for students;
- * improvement in schools' involvement with students' parents and families to create a consistent climate for learning as well as to empower parents to be advocates for their children's education;
- * development of a "safety net" for students through academic enrichment programs that provide extra academic support;
- * formation of school-community partnerships that include links with colleges and universities, the business community, and community-based organizations; and
- * use of student course enrollment and achievement data broken down by ethnic group and gender to monitor progress toward reform goals.

logically, and talk about the ways they are solving problems. Teachers act as facilitators, encouraging cooperation, motivating students in future careers and everyday life. All Family Math activities underscore the NCTM Standards and complement the school mathematics curriculum. Developed by EQUALS at Lawrence Hall of Science, University of California, Berkeley, Family Math was introduced to New Jersey schools by the Rutgers University Consortium for Educational Equity in 1985. Three-day workshops are offered by the Consortium for teachers and parents who want to conduct Family Math courses in their school. Other parent involvement programs include Family Science and Family Tools & Technology. For further information call: (908) 445-2071.

⁵⁰Partners for Reform in Science and Math (PRISM) videos and outreach sheets show parents how to become involved in their children's mathematics and science education. PRISM is a collaboration between the National Urban League and Thirteen/WNET. To order materials call the Annenberg/CPB Math and Science Collection at 1-800-965-7373.

⁵¹ For further information about EQUITY 2000, call (212) 713-8268.

Extracurricular activities such as math clubs^{52 53}, math teams^{54 55}, and other math activities⁵⁶ can also be used to encourage students to strive for excellence in mathematics.

Identifying Equity Concerns in Districts and Schools

In order to address the issues raised in this chapter, each district, each school, and each individual educator needs to reflect on the issues in conjunction with a realistic appraisal of their situation. Most of the suggestions in this chapter are addressed to teachers; this section, however, is addressed to school and district administrators.

Identifying Equity Concerns in Districts

In order to assure that we are providing an equitable learning environment for all students, we must review all district policies and practices which have an effect on equity. Each district should review the following questions, and consider the consequences, intended or unintended, of its policies and practices in each of these areas:

Administrative Policies and Practices

- * What are district policies on equity issues in general, and mathematics education in particular, including such areas as employment, school and classroom practices, student treatment, etc.?
- * Has the district discussed and adopted an explicit statement on equity?
- * What are district policies on tracking and grouping students in mathematics?
- * What are district referral and classification practices for special education?
- * What are district practices and expectations regarding various groups of children — gifted, basic

⁵² A Math Club in your school provides an opportunity for students to pursue their interests in mathematics beyond the curriculum. Even some elementary schools offer after-school activities for interested students. One way to start is to offer a challenging problem to students in your grade or post it in all the math classes. Students may be encouraged to submit solutions and the solutions can be presented and discussed, with prizes for correct solutions, at the first meeting of the Math Club! For information about math clubs and the over 50 math contests that are available each year to stimulate your “mathletes”, contact David Marain, Sparta High School, 70 West Mountain Avenue, Sparta, NJ 07871 or email dmarain@ix.netcom.com or call (201) 791-3118 (PM).

⁵³ MATHCOUNTS is a nationwide program that promotes math excellence among junior high school students and helps them and their parents become aware of career opportunities in math. Funded by the National Society of Professional Engineers and business and industry, MATHCOUNTS also sponsors a contest for “mathletes”. For information, call (703) 684-2828.

⁵⁴ Enter your high school in the New Jersey High School Math Contest and encourage your students to participate. Sponsored by the Association of Mathematics Teachers of New Jersey, the contest is held each November. For information, call Mary E. Froustet at 908/686-2767.

⁵⁵ Enter your middle school in **Solve It**, a Middle Grades Mathematics League. For information, write to the League Director, William B. Moody, Solve It, University of Delaware, Dover, DE 19716-2901 or call (302) 831-1658.

⁵⁶ Students in grades 8-12 can participate in the Gelfand Outreach Program in Mathematics at Rutgers University, and work on monthly problem sets which are reviewed by Rutgers faculty and graduate students. For information, call Harriet Schweitzer at (908) 445-0669.

skills, compensatory education, language-minority, etc. children?

- * What are the very highest expectations of the school district, for those students who are most successful in mathematics, science, and technology? To what extent are such high expectations encouraged widely, and to what extent might opportunities to achieve them be artificially or unnecessarily limited?
- * How do individual school's student populations reflect the racial/ethnic composition of the district as a whole?
- * What happens to students after they complete school in the district?
- * Are funds specifically allocated to advance equity concerns?
- * Do teaching assignments in mathematics and other teaching and administrative areas reflect the racial/ethnic diversity of the larger community and provide role models for all students?
- * What kinds of professional development activities are available to teachers, administrators, and guidance counselors to sensitize them toward the integration of equity and diversity in mathematics, science, and technology education?
- * What kinds of professional development activities are available to teachers which reflect the mathematics and science standards?
- * What kinds of professional development activities are provided to teachers regarding instructional/learning styles, expectations, teacher/student interaction?
- * Are professional development activities available to guidance counselors which focus on expectations, equity, and career development activities to encourage students to consider careers in mathematics and science?

Curriculum, Instruction, and Assessment

- * Are curriculum materials for mathematics and other subject areas free of bias? Do they represent all groups and encourage the participation of all students?
- * Are curriculum materials and instructional practices aligned with the recommendations of the *New Jersey Mathematics Standards*? Are equity issues addressed in implementation?
- * Are a variety of teaching strategies in mathematics used?
- * To what degree are cooperative learning groups in mathematics used to encourage all students to be actively engaged?
- * What kinds of alternative assessment strategies are being utilized to assess student achievement in mathematics, science, and technology education?
- * What are the requirements in mathematics, science, and technology for students? Do the requirements differ for different populations of students?
- * What is the level of participation of underrepresented students in advanced and honors mathematics courses? What are the percentages for the traditionally-achieving students?
- * How are students selected to participate in advanced and honors mathematics and science courses? What criteria are used to make this selection?
- * Are guidance counselors involved in conveying to students the importance of mathematics and science education? In what specific ways do they reach out to underrepresented groups of students to encourage them to pursue courses and careers in mathematics and science?
- * How are students encouraged to see mathematics and science as integral to the development of all

cultures and civilizations?

- * How is the relevance of mathematics to all careers demonstrated?

Community Outreach

- * Have parents been involved in the district's mathematics and science reform efforts?
- * Does the district offer opportunities for parents to learn about the importance of mathematics and science education, and to enable them to assist their children in learning these subjects?
- * What kinds of outreach strategies have been targeted to parents to educate them about the kinds of mathematics, science, and technology courses and programs that are available to students in the district?
- * In what specific ways have local business, industry, and community organizations been involved in district mathematics and science reform efforts?

Identifying Equity Concerns in Schools

Teachers and supervisors do not have much control over the areas enumerated above, especially policy areas where the responsibility belongs to the district. But those of us who are teachers and supervisors do have a lot of control over what happens in our classrooms and in our school buildings.

In order to identify equity concerns in our schools, a first step might be to take a good hard look at what is happening in the school. We should start by collecting data to ascertain how many students from each group are enrolled in the various courses offered by the school.

Such data, disaggregated by relevant groups, might include:

- * enrollment data for high school mathematics and science classes;
- * numbers of students in various tracks at various grade levels, including gifted and talented programs and special education; and
- * achievement data including grades and test scores in mathematics and science.

For instance, you might want to review enrollments in 12th grade mathematics courses; you should go back a year or two in collecting data and use a table like the one below (with additional rows and columns as appropriate) which presents information by gender and ethnicity.

Such tables will provide information to answer questions like the following:

- * How many students are not taking any math courses in their Senior year?
- * To what extent are various groups underrepresented in advanced math courses?
- * Are some groups underrepresented among students who take math in their senior year?
- * Are some groups overrepresented among students who enroll in senior year math courses but fail to complete them satisfactorily? (To respond to this question, you would need two tables, one recording numbers of those who register and the other recording numbers of those who complete the various courses.)
- * What is the selection process for admission to advanced math classes? Does it allow too few students in? Can students elect to take these courses or do they have to be selected? Can students

take multiple advanced courses or must they choose between mathematics and another discipline?

- * What are the grades of students from different groups?
- * How many girls and boys move “up a level” (e.g., from average to above average) in math and science courses each year? How many move “down a level” (e.g., from honors to above average)?

Number of Students	African American		Asian		Hispanic		White	
	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys
AP-Calculus								
Calculus (non AP)								
Pre-calculus								
Consumer Math								
Basic Skills Math								
General Math								
Discrete Math								
Prob./Stat.								
No math course								
TOTAL STUDENTS								

In other areas, the data to review would be suggested by some of the questions in the previous section on district responsibility. For instance are some groups more likely than others to be classified as learning disabled? Are factors unrelated to educational need encouraging these differential assignments? Are “special education” children getting the levels of mathematics they are capable of?

Once problems are identified, the next step should be to develop a plan for corrective action which will include a review of policies and practices which may have had the effect of discouraging the achievement of some groups. Suggestions for overcoming the barriers to equity in our schools are provided in the preceding section of this chapter.

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On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

http://www.rbs.org/eisenhower/res_equity.html/

The Mid-Atlantic Eisenhower Consortium for Mathematics and Science Education maintains a Web page on Equity Issues in Mathematics and Science.